Game Based Learning in Improving Students' Derivative Calculation Skills

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***Abstract*—This paper has investigated the effectiveness of video game based learning in improving at-risk students' derivative calculation. Video game applets in the derivative calculation are developed for the study, and APOS theory is applied to analyze the results. Finally, future work is discussed.**

Keywords—Calculus, Applets, Cognitive Learning, Mathematics Education, APOS, at-risk students

I. INTRODUCTION

STEM-related jobs grew at three times the rate of non-STEM jobs between 2000 and 2010. According to the report by Pulitzer Center on April 25, 2016, over 400,000 potential STEM Jobs were unfilled. By 2018, it is projected that 2.4 million STEM jobs will go unfilled [1]. On the other hand, a report by Mathematical Association of American (MAA) [2] called Americans' struggle with mathematics "the most significant barrier" to achieve a degree in both STEM and non-STEM fields. As a language of nature and gateway to Science, Technology, Engineering, and Mathematics (STEM), calculus has been taken by about 700,000 students every year in American colleges and universities, and the average DFW (grades D and F, and withdraw) rate is 27% [3]. In Historically Black Colleges and Universities (HBCUs), the failure rate is higher. Although African American and Latino students are 37% in all high schools, they constitute only 20% of students who take the AB calculus exam and only 11% of those who take BC exam, noticing that 67% of students who take calculus in colleges and universities have taken pre-calculus in their high schools [3].

Educators and researchers, among which, many are granted by organizations such as National Science Foundation (NSF) and MAA, have done a great research in calculus education during the past decades [2-8]. They investigated from textbooks to pedagogical methods, from curriculums to the fundamental focusses, and from students' diverse majors, educational backgrounds to the effect of high technology learning. One of the evident reasons is students' weak background in mathematic knowledge. About 50 percent of students fail to pass college algebra with a grade of C or above, by a report of MAA [2]. As far, we haven't found the national data on the percentage of students who take Calculus I with a grade C in algebra, but high failure rate in college algebra implies that students' weakness in college algebra is evident. Students who are at-risk in calculus generally have a weak background in prerequisite courses (college algebra and trigonometry).

Reform in Calculus education in colleges and universities becomes crucial.

Digital Game-Based-Learning (GBL) has been proposed in calculus learning in recent years. Educators, especially many serious GBL proponents have explored its effectiveness in Calculus learning, including the theories and the case studies [9-15]. Among them, perhaps the most convincing one is Triseum [16]. Using the commercial game, it applies innovative ways to inspire students in calculus learning. According to UB news on 1/10/2017, its preliminary testing results found that "79 percent of students playing Variant: Limits agreed that the game increased their knowledge of limits, and 76 percent of students playing Variant: Limits said the game was fun and engaging."

However, most efforts in GBL in mathematics have focused on mathematics from elementary to high schools [17]. Few works in high education have investigated [9, 15, 18, 19]. These works agree that Game-based learning in calculus can remove students' fear for abstract concepts and theorems, intrigue their interests, facilitate real problem-solving experience, turn passive learning to active learning, and get out of piecewise knowledge mire to an aerial view of knowledge. It helps students to prepare for their further major study well.

This study designs video game applets to help at-risk students in calculus derivative calculation. The goal is to investigate its effectiveness in GBL design for at-risk students to build up high-level mental constructions in calculus learning.

II. PEDAGOGICAL BACKGROUND

A. *Cognitive Psychology*

In his book Cognitive Psychology [20], Sternberg found that the distributing learning in variable contexts will improve the study result greatly. He suggested the possible answer is that "these diverse contexts help strengthen and begin to consolidate it."

It is also pointed out that a frequent repetition can fix mental associations more firmly in memory. Thus, repetition aids in learning. Finally, the role of "satisfaction" is the key to forming associations.

These cognitive learning rules are applied in our game-based learning design.

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B. APOS Theory

APOS Theory is a theory about mathematical understanding, its nature, and its development [21]. The theory and its application are based on the following two assumptions [21]:

1) The assumption of mathematical knowledge: An individual's mathematical knowledge is his/her tendency to respond to perceived mathematical problem situations and their solutions by reflection on them in a social context, and constructing or reconstructing mental structures to use in dealing with the situations.

2) The hypothesis on learning: An individual does not learn mathematical concepts directly. He/she applies mental structures to make sense of a concept. Learning is facilitated if the individual possesses mental structures appropriate for a given mathematical concepts is almost impossible.

Based on these two assumptions, APOS theory suggests that "an individual's understanding of a mathematical topic develops through reflecting on problems and their solutions in a social context and constructing or reconstructing certain mental structures and organizing these in schemas to use dealing with problem situations." The mental structures contain action, process, object, and schema. Dubinsky formulated suitable examples to clarify descriptions of these four steps [21]:

1) Action: A transformation is first conceived as an action when it is a reaction to stimuli which an individual perceives as external. It requires specific instructions, and the need to perform each step of the transformation explicitly. For example, a student who requires an explicit expression to think about the derivative of a function, $f'(x)$, where $f(x) = x^3$, and can do little more than performing the action $f'(x) = 3x^2$, is considered to have an action understanding of the derivative of a function.

2) Process: As an individual repeats and reflects on an action, it may be interiorized into a mental process. A process is a mental structure that performs the same operation as the action, but wholly in the mind of the individual. Specifically, the individual can imagine performing the transformation without having to execute each step explicitly. For example, an individual with a process understanding of the derivative of a function, say $g(x) = (x^2 + 1)^2$, will construct a mental process which could include that (x) should first be written in a simplified form by squaring the binomial $(x^2 + 1)$ and then the derivative can be found by applying the rule, the derivative of a sum of functions is the sum of the individual derivatives of functions.

3) Object: If one becomes aware of a process as a totality, realizes that transformations can act on that totality and can actually construct such transformations (explicitly or in one's imagination), then we say the individual has encapsulated the process into a cognitive object. For example, when finding the derivative of functions an individual may confront situations requiring him/her to apply various actions and/or processes. These could include thinking about a function as the composite of two functions, for example, the function $h(x) = (x^2 + 1)^{100}$ is the composite of the functions $f(x) = x^{100}$ and $g(x) = (x^2 + 1)$, since $h(x) = f\big(g(x)\big)$. To find the derivative, x should first be conceptualized as an object which comprises of the composite of two functions. To this function object, the process understanding for finding derivatives must be encapsulated in the context of the chain rule to find the derivative $h'(x)$.

4) Schema: A mathematical topic often involves many actions, processes and objects that need to be organized and linked into a coherent framework, called a schema. It is coherent in that it provides an individual with a way of deciding when presented with a particular mathematical situation, whether the schema applies. For example, the coherence might lie in understanding that to determine the local extrema of a function, say $h(x) = (x^2 - 1)^{100}$, the following must be considered: the derivative $h(x)$, the critical points of $h'(x)$ occur where $h'(x)$=0. These critical points should be used to construct the sign diagram of $h'(x)$, and this should be analyzed to determine the nature of the extrema of h.

C. The ACGE teaching cycle

In 1996, Asiala et al. proposed a teaching cycle called ACE [9]. It represents a mathematical learning cycle of Activities, Classroom discussion, and Exercise outside of class. The cycle is based on APOS theory. From ACE, we proposed ACGE cycle in calculus derivative learning, which adds game based learning before the exercise.

Activities are designed to help students to construct mental structures of APOS. Students will be guided to think about the activities and its relation to the mathematical concepts. The next step is to understand the mathematical meaning in the in-class exercise. Students discuss their exercise results in the group and with the teacher. After the class, students play video games on the topic they work on in the class. The games help them to knit what they learn into their mathematical knowledge network and clear up the possible confusion by game tutorial and by game-play exercise. Finally, students solve standard problems related to the topic being studied. The repeating ACGE teaching cycle helps students to make mental constructions and learn their mathematical subjects.

D. Research framework

Asiala, et al. proposed a basic research framework based on APOS theory [9] and we adopt the framework for the study

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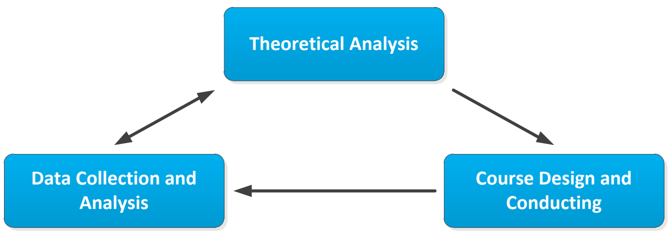
(Fig. 1). Three steps are designed to conduct the study: theoretical analysis, course design and conduct, and data collection and analysis. Two sets of measurement are given to collect the data. Mastery exams (MEs) are given to select at-risk students in the derivative calculation and to measure their performance after the study. MEs investigate the overall possible effectiveness of video game learning in their derivative calculation learning. Two specific derivative questions are given before and after the study, for the purpose of investigating mental constructions levels in APOS theory.

Fig. 1. Research Framework.



III. METHODOLOGY

The goal of the study is to investigate the effectiveness of video game-based learning in the derivative computation for at-risk students. The research framework follows the module in the section II.D, the game design follows the rules in cognitive learning in II.A, the experimental learning cycle is ACGE in the section II.C, and the result analysis will be conducted based on APOS in II.B.

A. *Experiment Design*

This study aims at helping at-risk students in calculus I study specifically. The program lasts three weeks. We design the following steps in the experiment:

Step 1

Recruit the students who are at-risk in derivative computation.

Mastery Exam is a screen-test to find at-risk students. Students who fail to pass ME are sent to the lab, where they will accept a one-on-one tutoring by instructors. After more practice, they are given an ME again. Students who fail to pass the second ME will be recruited to our game-based learning program.

Step 2

Select the applet(s) for each student according to his/her ME performance.

For each of students in the program, we first analyze his/her two ME papers and diagnose the type(s) of common problems each student has. Based on that, the specific applet(s) will be chosen for the student to play and learn in the lab. We category students' common mistakes in derivative calculation into the following four problems:

a) Unable to identify the composite functions successfully

b) Failed to mesmerize the formulas correctly

c) Unable to apply the chain rule successfully

d) Weak computation background in college algebra and trigonometry

Step 3

When participants arrive the lab, a quiz is first given with three random derivative formulas. Then a specific applet will be assigned to him/her to play. The game contains two parts: tutorial and gameplay. The participant will finish all the scenarios or until one hour is gone. Students will come back to the lab until finish all the applets assigned to him or her.

Step 4

Three weeks later, a third ME test will be given to all students who fail to pass the previous two MEs, including both participants and none-participants.

Step 5

Analyze data based on APOS theory.

B. *The Game: Cala*

The game developing team includes two computer science major students in UMES. Game applets are considered as the video game learning means for the following reasons:

1) The game applets are accessible for college students. EDUCAUSE Report has found that in 2014, over 84% undergraduate students own smartphones [22], and we'll expect even a higher percent at present.

2) Applets fit our goal better than an integrated video game. To at-risk students, an applet is easier to focus on one specific problem at a time and to attain an achievement before move on.

3) Compared with developing a big commercial game, an applet is easier to develop and much faster to put to use. It is well-suited to our campus community because the pool of possible programmers is small.

4) Applets could be integrated into one big project in the future as the study will be conducted for further research.

The game is called Cala. So far we have developed four scenarios corresponding to four different categories of common mistakes which are listed in step 2 of section III.A. Scenario 1 helps students to identify a composite function. One of the biggest challenges for our students in derivative computation is to identify the composite functions such as $y = \sqrt{\sin(3x)}$ or $f(x) = \frac{1}{\tan\sqrt{e^{3x}}}$. At-risk students simply identify them as trigonometry functions. We design a scenario of Russian Doll to help them identifying the composite functions in nested layers from the most outside function to the inner ones. It is followed by an exercise that in a garden with nested functions as plants, a rabbit named Cal comes out to each them. The player must choose the correct function identity to protect the plant. In Problem 2, the applet applies songs and cartoon characters to help students make both visual and sound connections with each derivative formula. It also designs a story to point out the common mistakes made by students in memorizing formulas. A flip out game is designed for students to match formulas.

Problem 3 is the most serious for students. In APOS theory, students with problem 3 have mental constructions developed up to the action level, but the action fails when they are given a complicated composite function. This is because they either have problem 1 (difficulty in identifying composite structure), or they need profound understand and exercise in understanding chain rule. The latter case shows that students are at the first two steps of APOS in II.B. Based on Scenario 1, Scenario 3 uses an animation to show how to compute derivative of such a nested function with the chain rule. In addition, we apply reward rule greatly in building up a connection between a composite function and chain rule, based on the principle that "the role of 'satisfaction' is the key to forming associations" in section 2.6 [Stanberg 2014]. One of the most common mistakes students have in misusing the chain rule is to interpret formula $(f \circ g)'_{(x)} = f'_{(g(x))} \cdot g'_{(x)}$ as the equivalence of $(f \circ g)'_{(x)} = f'\big(g(x)\big)$. To remove such a wrong connection, we design a story that $g'(x)$ breaks into the house of f' and takes over the position of $g(x)$. Last but perhaps the most challenging part is Problem 4. Students with weak background make various mistakes which are rooted back to college algebra and trigonometry. Scenario 4 focuses on a few common mistakes students have: fraction computation, grouping symbol, distributing the minus sign, and simplification.

Two scenario screenshots are attached in Fig. 2 and Fig. 3.

Fig. 2. Find the correct answer before the rabbit eats out the plant.

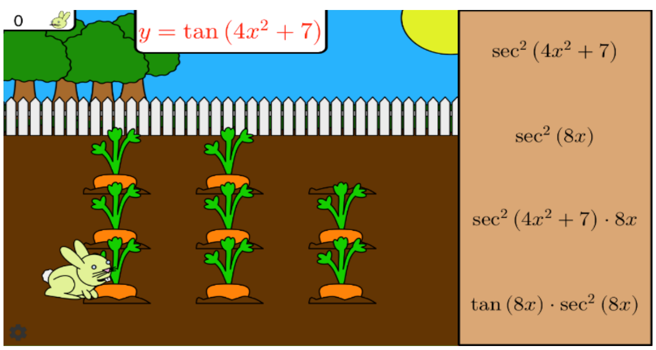
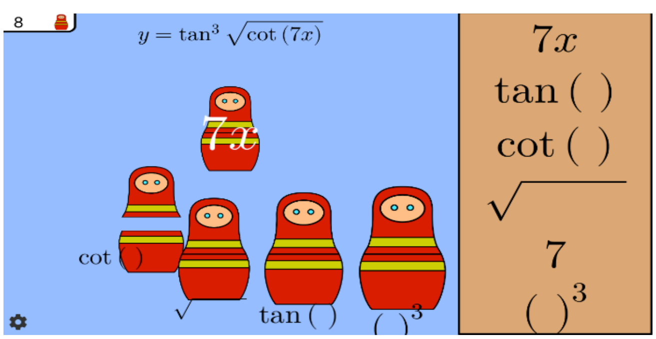


Fig. 3. Russian dolls for composite functions



IV. Measurements, Results & Analysis

A. Measurement

Mastery Exam is designed to measure students' ability in derivative calculation before and after the study. It has eight basic derivative problems, testing the basic derivative formulas, basic derivatives rules especially chain rule, and fundamental computation skills. Students must get seven out of eight problems perfectly to pass it. The first two MEs help to find out the candidate students who are at-risk to pass the course. The 3rd ME is to measure if game based learning helps to improve participants' derivative computation abilities, by comparing two groups of students' performance: one is with the aid of game-based learning and the other one is not.

We measure not only the quantity of students who passed, but the mental constructions of the same group before and after video game learning is also analyzed based on APOS theory. An investigation is made to analyze if the mental constructions of the student have developed up to a higher level with the aid of game-based learning in the derivative calculation.

B. Results

1) ME Tests

The candidates in this study were 108 students at University of Maryland Eastern Shore in Spring 2017. Among them, 103 students (97%) major in STEM, 35 students (32.4%) retook the course, and 51 students (47%) had C in prerequisite courses (college algebra and/or trigonometry).

Only 16 students (14.8%) passed 1st ME, and 92 (85.2%) students failed. Next, among the 92 students who failed in 1st ME, 89 (96.7%) took the 2nd ME, and 14 (13%) passed. As far, 78 students (72%) did not pass MEs. We recruited 30 students for our study. Among them, 28 participants had C and the rest 2 students had S (Satisfied) in their prerequisite courses, and 17 out of 30 participants (57%) were retaking Calculus I. We chose more retaking students into the study, because we wish to study how video game learning would help these students to remove their wrong connections in their mental constructions, and build up a new correct one. Also, our program focuses on students with weak math background.

In 3rd ME, there were total 57 students took the exam. Besides 30 participants, 27 students are volunteers who failed the previous two MEs but did not participate in the study. 17 out of 30 participants and 6 out of those 27 non-participants passed 3rd ME. Among those participants who passed 3rd ME, 8 are retaking the course.

2) APOS analysis problems

Before and after the study, Q1 and Q2 were given to 30 participants to answer respectively.

- Q1. Find the derivative of the following function:

  $f(x) = \sqrt{\cos(2x)}$

A. $\sin(2x)$ B. $-2\sin 2x$ C. $\frac{1}{2}(\cos 2x)^{-\frac{1}{2}}$

D. $\frac{1}{2}(\sin 2x)^{-\frac{1}{2}}$ E. $\frac{1}{2}(\cos 2x)^{-\frac{1}{2}}(-2\sin 2x)$ F. None of them

TABLE I. QUESTION 1 ANALYSIS OF STUDENTS CHOICES $(n = 30)$

Q 1	Total	A	B	C	D	E	F
Participants	30	3	9	4	7	3	4

- Q2. Find the derivative of the function:

  $f(x) = \cot\big(\sin(3x)\big)$

A. $-\csc^2\big(\sin(3x)\big)$ B. $-\csc^2(\sin 3x) + \cot(\cos 3x)$

C. $-\csc^2\big(\cos(3x)\big)$ D. $-\csc^2\big(3\cos(3x)\big)$

E. $-3\csc^2\big(\mathrm{si}(3x)\big)\cos(3x)$ F. None of them

TABLE II. QUESTION 2 ANALYSIS OF STUDENTS CHOICES $(n = 57)$

Q 2	Total	A	B	C	D	E	F
Participants	30	1	2	3	7	17	0
Non-participants	27	1	4	6	8	6	2

C. *Analysis*

In 3rd ME, the passing rate for participants is 56.7%, and the one for non-participants is 22%. This may or may not prove that game based learning did improve students' derivative calculation ability because participants spent extra hours in computed-aid study. However, if we compare the performance of the same group of participants before and after the study, with the help of APOS theory, we might see something different.

Let's look at the results of APOS analysis problems in the previous section. Question 1. 12 students chose A and B. This suggests that about 40% of students had mental constructions developed up to the action level, but failed to identify the type of composite function. Another 11 students (36.7%) who chose choice C or D did identify that it is a composite function, and had mental construction to the process level, but failed to identify the function as the composite of three functions $y = f\Big(g\big(h(x)\big)\Big)$ where $f(x) = \sqrt{x}$, $g(x) = \cos(x)$ and $h(x) = 2x$, respectively. The number of choice E suggests that 3 out of 30 students (10%) students had mental construction up to the object level. Choice F shows approximately 13.3% of students have no idea of the basic derivative technique.

In Question 2, one student (3.3%) chose A. It meant that the student could do some derivative calculation but failed to memorize the derivative formula correctly. The student also failed to identify its composite property. Choice B shows that two students (6.7%) could do some basic derivative calculation, but failed to identify the nested structure of the function. Misunderstanding it as a product of a cotangent function and a sine function, these two students applied product rule to the problem-solving. Students who chose C and D identified the given function as a composite function but failed to interpret the nested structure precisely. Once again, Choice F tells that no student (0%) had no idea of the basic derivative technique. Students chose Choice A and B had mental construction up to the level action. 17 students found the correct answer E. In a summary, 10% of students had mental constructions up to action level, about 33.3% of students had mental constructions up to processing level, and about 56.7% of students had mental constructions up to the object level.

APOS theory tells us that a learning processing is a process that mental constructions keep on moving, from the action, to the process, to the object, and to the schema level. If students' mental constructions are upgrading, their cognitive learning is progressing. Therefore it is important to compare their cognitive progress of the same group. Table 3 gives the comparison of participants' mental constructions levels before and after the study.

TABLE III. APOS MENTAL CONSTRUCTION LEVELS BEFORE AND AFTER THE STUDY $(n = 30)$

	Action level	Process leve	Object level	No connection
Before	40.0%	36.7%	10.0%	13.3%
After	10.0%	33.3%	56.7%	0.0%

V. CONCLUSION AND FUTURE WORK

- The result shows that all students had built up certain mental constructions after the study. It suggests that video game-based-learning helps students to build up certain mental constructions who had none before.
- After the study, 30% of students upgraded their mental constructions from the action level to higher ones, and 13% at process level upgraded to object one.
- Among 30 participants, 17 were retaking the course and 13 were the first time to take the course. If we compare these two groups, we find that after the study, the passing rate for the retaking group is 47% (8 out of 17 passed) and that for the first-time-taking group is 53% (9 out 13). Does this suggest that video game learning does not help those retaking students as much as the others? Or it is because the games were not specific enough for them to remove their wrong mental constructions in the knowledge? If it is, how to use video game means helping those retaking students to remove their wrong connections in knowledge and replace with the new correct ones?

There are a lot of questions we need to ponder. Our sample size is small, evidently. Extending the experimental sample size will help us to get a better vision. Maharaj points out that many students perform poorly because of (1) Insufficient to handle information in a symbolic form (2) Lack adequate schema or frameworks to organize the link among different objects [23]. Video games have strength in visualizing a symbolic form, and vividly make a link between different objects. Necessary ingredients of video game design should be investigated for effective mental constructions. Games should meet specific needs for students with a different background. For example, for students who retake calculus course, one of their challenges is how to remove wrong connection in their math knowledge network, and build up correct ones. Would games be helpful in the issue? How? These will be our future work.

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